

Moments of generalized parton distribution functions and the nucleon spin contents

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Abstract

It is shown that, based only on two empirically known facts besides two reasonable theoretical postulates, we are inevitably led to a conclusion that the quark orbital angular momentum carries nearly half of the total nucleon spin. We also perform a model analysis to find that the quark spin fraction $\Delta\Sigma$ is extremely sensitive to the pion mass, which may resolve the discrepancy between the observation and the prediction of the recent lattice QCD simulation carried out in the heavy pion region.

The so-called “nucleon spin puzzle” raised more than 15 years ago is still an unsolved fundamental puzzle in hadron physics [1],[2]. If intrinsic quark spin carries little of the total nucleon spin, what carries the rest of the nucleon spin ? It is the question to be answered. Admitting that the QCD is a correct theory of strong interaction, the answer must naturally be sought for in the following three ; the quark orbital angular momentum (OAM), the gluon polarization, and the gluon orbital angular momentum.

Roughly speaking, there exist two contrasting or opposing standpoints to try to answer the above question. The chiral soliton picture of the nucleon emphasizes the importance of the quark orbital angular momentum [3],[4]. On the other hand, the possible importance of the gluon polarization was stressed by several authors in relation with the axial anomaly of QCD [5],[6],[7]. Later, the role of QCD anomaly was understood more clearly within the framework of the perturbative QCD, especially in view of the factorization-scheme dependence of parton distribution functions [8]–[10]. Nonetheless, the serious problem is that no one can give any reliable theoretical prediction for the actual magnitude of Δg .

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An important remark here is that it is meaningless to talk about the nucleon spin contents without reference to the energy scale of observation. In fact, it is a widely known fact that the gluon polarization grows rapidly as Q^2 increases, even if it is small at low energy [11]. In contrast, the gluon orbital angular momentum decreases rapidly to partially compensate the increase of Δg . (Strictly speaking, these statements are gauge dependent, since it is known that there is no gauge invariant decomposition of gluon angular momentum into spin and orbital angular momentum [12].) Consequently, when we talk about the nucleon spin contents naively, we should implicitly understand that we are thinking of it at low energy scale of nonperturbative QCD. At this low energy, the CQSM predicts that [4],[13],[14]

$$\Delta\Sigma \simeq 0.35, \quad 2L_q \simeq 0.65, \quad (1)$$

which means that the quark OAM dominates over the contribution of quark intrinsic spin.

We repeat the question, “Which carries the rest of the nucleon spin, L_q or Δg ?” Naturally, only experiments can answer it. A direct measurement of Δg via photon-gluon fusion processes is one of the most promising direction of study. For instance, the Compass group recently extracted the value of $\Delta g/g$ through the analysis of the asymmetry of high p_T hadron pairs. Their first result for $\Delta g/g$ has turned out to be fairly small [15],

$$\Delta g/g \sim 0.06 \pm 0.31(stat.) \pm 0.06(syst.), \quad (2)$$

although it is premature to draw any decisive conclusion only from this analysis. On the other hand, the key quantity for the direct measurement of J_q or L_q is the generalized parton distributions appearing in the cross sections of deeply virtual Compton scattering and/or deeply virtual meson productions [16]–[18]. What plays the central role here is Ji’s quark angular momentum sum rule.

Here, we start our argument with the familiar definition of the generalized form factors $A_{20}(t)$ and $B_{20}(t)$ of the nucleon, which is given as a nonforward matrix element of QCD energy momentum tensor $T_{q,g}^{\mu\nu}$:

$$\langle P' | T_{q,g}^{\mu\nu} | P \rangle = \bar{U}(P') \left[A_{20}^{q,g}(t) \gamma^{(\mu} P^{\nu)} + B_{20}^{q,g}(t) \frac{P^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha}{2M} \right] U(P) + \dots \quad (3)$$

According to Ji’s sum rule, the total angular momentum carried by quark and gluon fields in the nucleon is related to the forward ($t = 0$) limit of these generalized form factors as [18]

$$J^{u+d} = \frac{1}{2} [A_{20}^{u+d}(0) + B_{20}^{u+d}(0)], \quad (4)$$

$$J^g = \frac{1}{2} [A_{20}^g(0) + B_{20}^g(0)]. \quad (5)$$

Remembering the fact that the above generalized form factors $A_{20}^{u+d}(0)$ and $A_{20}^g(0)$ are related to the second moments of the unpolarized generalized parton distribution functions of quarks and gluons, which reduce to the familiar unpolarized distributions for quarks and gluons in the forward limit, they just represent the the total momentum fraction of quarks and gluons in the nucleon as

$$A_{20}^{u+d}(0) = \int_0^1 x [u(x) + \bar{u}(x) + d(x) + \bar{d}(x)] dx \equiv \langle x \rangle^{u+d}, \quad (6)$$

$$A_{20}^g(0) = \int_0^1 x g(x) dx \equiv \langle x \rangle^g. \quad (7)$$

On the other hand, the second B parts are sometimes called the anomalous gravitomagnetic moments (AGM) of the constituents of the nucleon [19]–[21]. From the conservation of total momentum and angular momentum, it follows that

$$A_{20}^{u+d}(0) + A_{20}^g(0) = 1, \quad (8)$$

$$A_{20}^{u+d}(0) + B_{20}^{u+d}(0) + A_{20}^g(0) + B_{20}^g(0) = 1, \quad (9)$$

which in turn dictates a nontrivial identity :

$$B_{20}^{u+d}(0) + B_{20}^g(0) = 0. \quad (10)$$

To proceed further, we must distinguish three possibilities below :

1. $B_{20}^{u+d}(0) = -B_{20}^g(0) \neq 0$,
2. $B_{20}^{u+d}(0) = B_{20}^g(0) = 0$,
3. $B_{20}^u(0) = B_{20}^d(0) = B_{20}^g(0) = 0$.

The recent lattice QCD simulation by LHPC Collaboration gives a strong support to the second possibility that the total quark contribution to the nucleon AGM vanishes [22],[23]. This happens as a cancellation of the u - and d -quark contributions, i.e., $B_{20}^u(0)$ and $B_{20}^d(0)$, which have sizable magnitudes with opposite signs. Noteworthy here is the fact that both of $B_{20}^u(0)$ and $B_{20}^d(0)$ have fairly strong dependence on the pion mass but their sum is almost independent on it. In any case, this lattice analysis seems to deny the third possibility indicated in [19] on the basis of the equivalence principle, but strongly supports the second possibility, which is the basis of the following argument. In fact, once we accept this postulate, we are led to a surprisingly simple result that the total quark angular momentum is just a half of the total quark angular momentum fraction [22],[23],[19],[20] :

$$J^{u+d} = \frac{1}{2} \langle x \rangle^{u+d}. \quad (11)$$

Now, we can go further. First, let us recall an empirically well-accepted understanding that, even at low energy scale like $Q^2 \simeq (600 \text{ MeV})^2$, the gluon field seems to carry about $(20 \sim 30)\%$ of the total nucleon momentum. (The widespread belief that the quark and gluon fields share equal amounts of nucleon momentum applies to the asymptotic case of large Q^2). For instance, one may consult the well-established GRV fit of the unpolarized parton densities [24]. (See also [25].) Their next-to-leading order fit of the gluon density is given at $Q^2 = \mu_{NLO}^2 = 0.40 \text{ GeV}^2 \simeq (630 \text{ MeV})^2$ as

$$xg(x, \mu_{NLO}^2) = 20.8 x^{1.6} (1-x)^{4.1}. \quad (12)$$

This turns out to give

$$\langle x \rangle^g \equiv \int_0^1 xg(x, \mu_{NLO}^2) dx \simeq 0.30. \quad (13)$$

Conversely saying, we can say that, at low energy, the quark field carries at least $(70 \sim 80)\%$ of nucleon momentum, which in turn must be equal to the total quark angular momentum fraction, according to the aforementioned argument, such that

$$2J^{u+d} = \langle x \rangle^{u+d} = (0.7 \sim 0.8). \quad (14)$$

On the other hand, through the analysis of polarized deep-inelastic scatterings, we already know that the intrinsic quark polarization $\Delta\Sigma$ is about $(20 \sim 35)\%$ (see, for instance, the recent review [26]) :

$$\Delta\Sigma \simeq (0.2 \sim 0.35). \quad (15)$$

Putting these two observations (14) and (15) together, we find that the quark orbital angular momentum fraction is nearly 50%,

$$2L^{u+d} = 2J^{u+d} - \Delta\Sigma \simeq 0.5. \quad (16)$$

That is, once admitting that the isosinglet combination of the quark contribution to the nucleon AGM vanishes, we are inevitably led to a surprising conclusion that the quark OAM carries nearly half of the nucleon spin, only with use of the empirically known information.

One might wonder why our conclusion is entirely different from that obtained by the LHPC Collaboration [23],[23], who claims that the quark OAM is negligibly small, in spite that our argument above is based on a result of the LHPC group, i.e., $B_{20}^{u+d}(0) = 0$. The rest of the present report is devoted to clarifying this point. The reason can easily be traced back to the fact that, instead of using the empirical value of $\Delta\Sigma$, they used their theoretical predictions for it,

$$\Delta\Sigma(\text{LHPC}) \simeq 0.682, \quad (17)$$

which is fairly large and clearly contradicts the observation. Why does their analysis give very large $\Delta\Sigma$, then? This is probably because their simulation was performed with quite large pion mass around $m_\pi \simeq (700 \sim 900)$ MeV, which is far from our realistic world close to the chiral limit. As we shall discuss below, the strong sensitivity of $\Delta\Sigma$ on the pion mass seems to be a likely solution to the above-mentioned discrepancy.

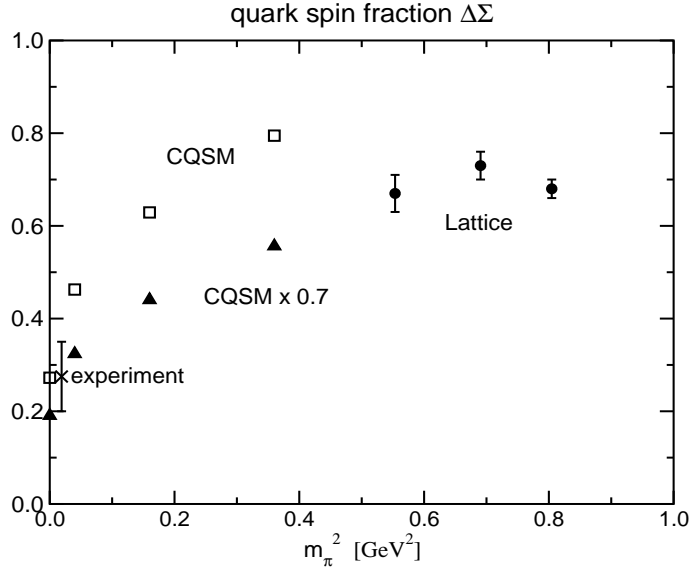


Figure 1: The quark spin fraction $\Delta\Sigma$ of the nucleon in dependence of the pion mass m_π^2 [GeV 2]. The filled circles with error bars represent the predictions for $\Delta\Sigma$ by the LHPC group, corresponding respectively to $m_\pi = 744, 831$, and 897 MeV, while the cross with error bar stands for the empirical value corresponding to the physical pion mass $m_\pi = 138$ MeV. The predictions of the CQSM are shown by the open squares for four values of m_π , i.e., $m_\pi = 0, 200, 400$, and 600 MeV. Also shown by the filled triangles are the predictions of the CQSM scaled by the factor 0.7 .

Now, we shall show it on the basis of the CQSM [27],[4]. Within the framework of the CQSM, we first solve the mean-field equation of motion self-consistently for several values of m_π . The model is defined with a physical cutoff. Here we use the Pauli-Villars regularization scheme with double subtraction terms [28]. The relevant regularization parameters are all fixed uniquely from reasonable physical requirements. How to introduce finite pion mass into the whole scheme is explained in [28]. Here, we tried to find a self-consistent soliton profile with the fixed value of the dynamical quark mass $M = 400$ MeV. This is repeated for several values of pion mass, i.e., $m_\pi = 0, 200, 400$, and 600 MeV. In this analysis, no stable solution was found for $m_\pi > 620$ MeV. We then evaluate $\Delta\Sigma$ for each soliton solution with different value of m_π . The results are shown in Fig.1 together with the predictions of the LHPC Collaboration [22],[23]. The filled circles with error bars represent the predictions

for $\Delta\Sigma$ by the LHPC group, corresponding respectively to $m_\pi = 744, 831$, and 897 MeV , while the cross with error bar stands for the empirical value corresponding to the physical pion mass $m_\pi = 138 \text{ MeV}$. The predictions of the CQSM are shown by the open squares for four values of m_π , i.e., $m_\pi = 0, 200, 400$, and 600 MeV . One clearly sees that $\Delta\Sigma$ is very sensitive to the value of m_π , especially when approaching the chiral limit $m_\pi \rightarrow 0$. Inspired by the indication of the GRSV fit, which dictates that the quark fields carries only 70% of the total nucleon momentum and also the total spin, one may tentatively renormalize the predictions of the CQSM by multiplying a factor of 0.7. The results are shown by the filled triangles. It is interesting to see that these points appears to be smoothly connected to the lattice predictions given in the large m_π domain. We hope that the lattice simulation in the near future will be extended to the region of smaller m_π and that it will confirm the strong m_π dependence of $\Delta\Sigma$ predicted by the CQSM, although the lattice QCD would still need a help of some other theoretical technique like the chiral perturbation theory to explore the region of m_π very close to the chiral limit [29].

Summarizing our arguments, we have shown that, with use of the two empirical knowledge alone, aside from the two reasonable theoretical postulates, we are inevitably led to a drastic conclusion that the *quark orbital angular momentum carries nearly half of the total nucleon spin*. The two theoretical postulates here are

- Ji's angular momentum sum rule : $J^{u+d} = \frac{1}{2} [\langle x \rangle^{u+d} + B_{20}^{u+d}(0)]$,
- absence of the *net quark contribution* to the *anomalous gravitomagnetic moment* of the nucleon : $B_{20}^{u+d}(0) = 0$.

On the other hand, the two empirically known facts we have used are

- the fraction of the quark momentum and angular momentum of the nucleon at low energy scale, $Q^2 \simeq (600 \text{ MeV})^2$: $\langle x \rangle^{u+d} = 2J^{u+d} \simeq (0.7 \sim 0.8)$,
- the quark spin fraction from polarized DIS analyses : $\Delta\Sigma \simeq (0.2 \sim 0.35)$.

Although there remains some room concerning how to define the angular momentum of the constituents of the nucleon [30], it is reasonable to stick to Ji's definition, which leads to the above sum rule. Otherwise, we would lose a only clue to experimentally access the quark angular momentum in the nucleon. Thus, only one factor, which might potentially alter our conclusion, is the second postulate, i.e., $B_{20}^{u+d}(0) = 0$. Although it is strongly supported by the lattice simulation by the LHPC Collaboration, an independent check is highly desirable. Also desirable is an analytical proof of it within the framework of (nonperturbative) QCD.

We have also shown that the above-mentioned conclusion, *obtained independently of any models*, is qualitatively consistent with the predictions of the CQSM. The CQSM predicts very strong dependence of the quark spin fraction $\Delta\Sigma$ on the pion mass : it reproduces small $\Delta\Sigma$ in the domain close to the chiral limit, it also smoothly matches the predictions of the LHPC Collaborations obtained in the heavy pion region. It is hoped that this behavior of $\Delta\Sigma$ will be confirmed by the lattice simulation in the near future.

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